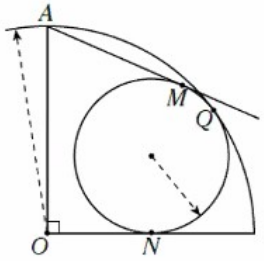
Grabando

CICLO REPASO UNI

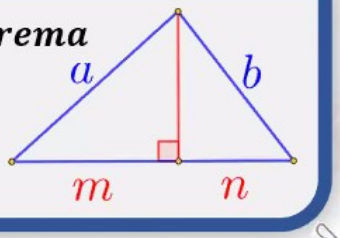
Problema 1

Según el gráfico, M, N y Q son puntos de tangencia y $ON=2$. Calcule AM.



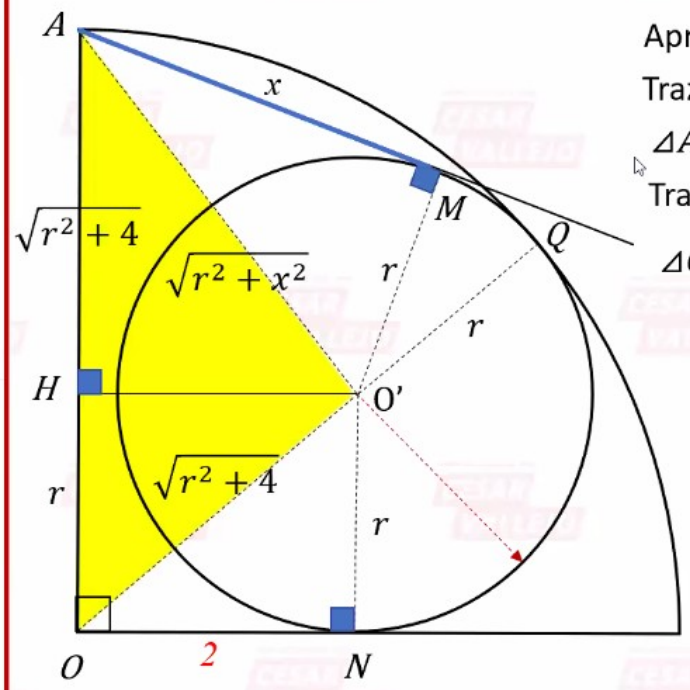
A) $3\sqrt{3}$ B) $3\sqrt{2}$ C) $2\sqrt{3}$
D) $2\sqrt{2}$ E) 4

Teorema



$$a^2 - b^2 = m^2 - n^2$$

Resolución



Piden: $AM = x$

Aprovechamos los puntos de tangencia M y N

Trazamos $\overline{O'M}$

$\triangle AMO'$: $AO' = \sqrt{r^2+x^2}$

Trazamos $\overline{O'N}$

$\triangle ONO'$: $OO' = \sqrt{r^2+4}$

Trazamos $\overline{O'H} \perp \overline{AO}$

Sabemos que O, O', Q son colineales

Entonces: $AO = AQ = r + \sqrt{r^2+4}$

$\Delta AO'O$ $(T.Proyecciones)$:

$$(AO')^2-(OO')^2 = (AH)^2-(HO)^2$$

Reemplazando:

$$(\sqrt{r^2+x^2})^2-(\sqrt{r^2+4})^2 = (\sqrt{r^2+4})^2-(r)^2$$

Operando: $x^2 = 8 \Rightarrow x = 2\sqrt{2}$

CLAVE: **D**

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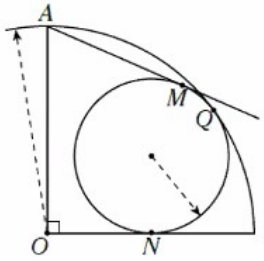
Problema 1

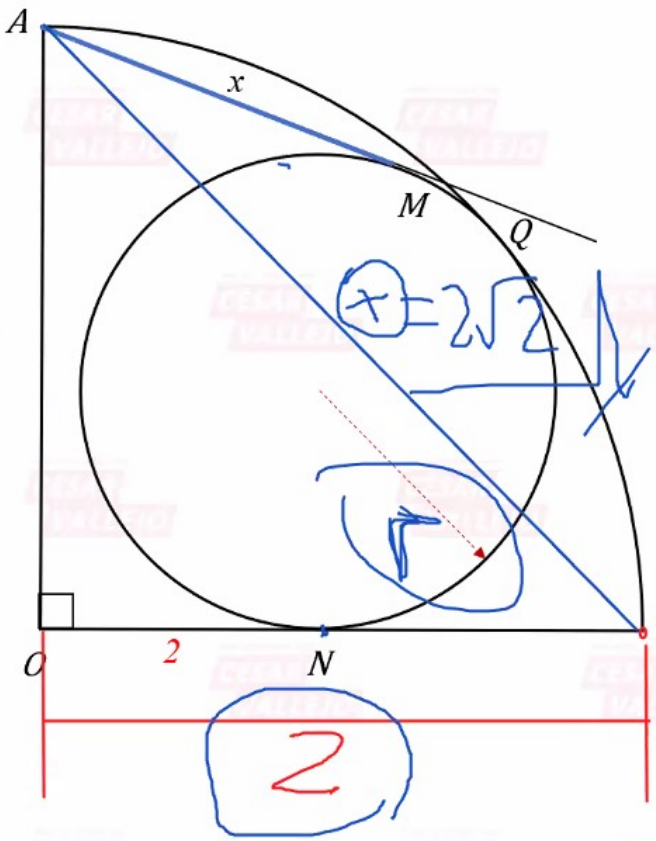
Según el gráfico, M, N y Q son puntos de tangencia y $ON=2$. Calcule AM.



A) $3\sqrt{3}$ B) $3\sqrt{2}$ C) $2\sqrt{3}$
D) $2\sqrt{2}$ E) 4

Resolución

Piden: $AM = x$



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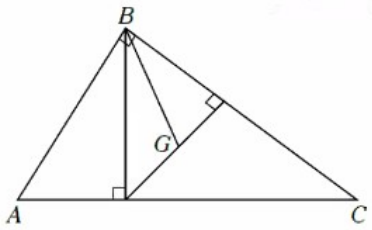
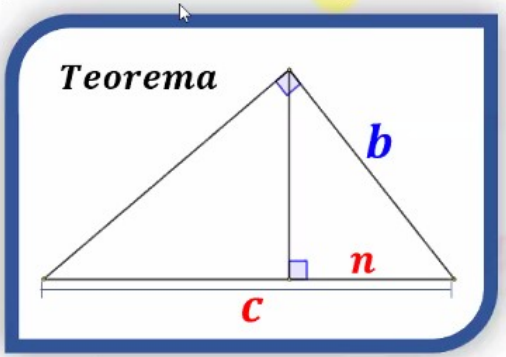
Problema 2

En el gráfico mostrado, G es el baricentro del triángulo ABC y $BC=6$. Halle BG.



A) 2 B) 3 C) $\sqrt{2}$

D) $\sqrt{3}$ E) $\sqrt{6}$



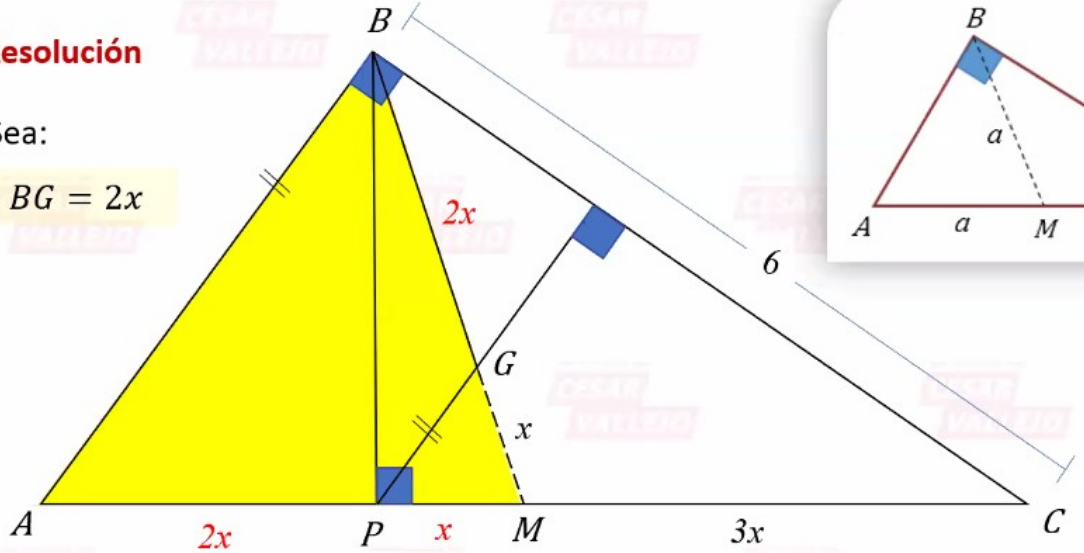
Cateto al cuadrado

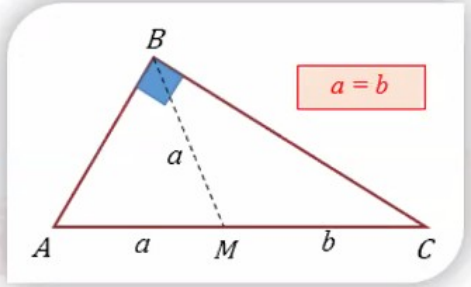
$b^2 = c.n$

Resolución

Sea:

$BG = 2x$





Prolongamos $\overline{BG}$ hasta M, Como G es baricentro, entonces $GM = x$, $\boldsymbol{AM} = MC = 3x$

$\Delta AMB\ (T.Thales)$: $\dfrac{AP}{PM} = \dfrac{2x}{x}$ ➡ $AP = 2PM = 2x$ Se sigue que: $PC = 4x$

ΔABC: $BC^2 = (CA)(CP)$

$6^2 = (6x)(4x)$

Finalmente: $2x = \sqrt{6}$

CLAVE: **E**

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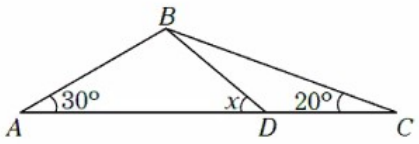
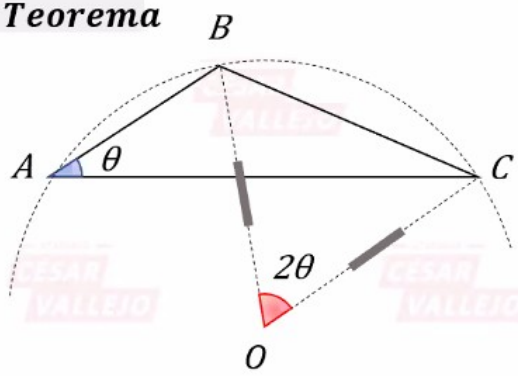
Problema 3

En el gráfico, $AD=BC$. Calcule x.

A) 60°
B) 50°
C) 70°
D) 40°
E) 30°

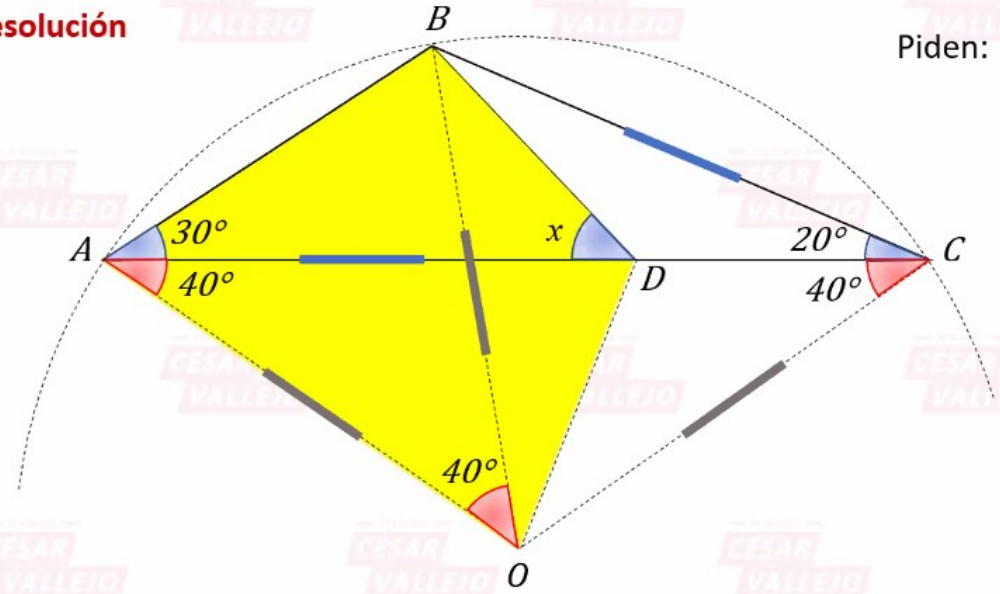


Teorema



O: Circuncentro

Resolución



Piden: $m\measuredangle BDA = x$

Construimos el triángulo equilátero BOC: OC = OB = BC

O resultaría ser el circuncentro del ΔABC, Trazamos el arco ABC, de centro O

$OA = OB = OC$, además : $m\measuredangle AOB = 40°$

como $AD = OB$, y $m\measuredangle OAD = m\measuredangle AOB = 40°$

$\square ABDC$: Trapecio Isósceles

Se sigue que: $\overline{BD} \parallel \overline{AO}$

Finalmente: $x = 40°$

CLAVE: **D**

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x

Grabando

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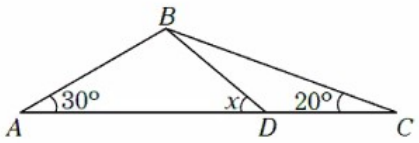
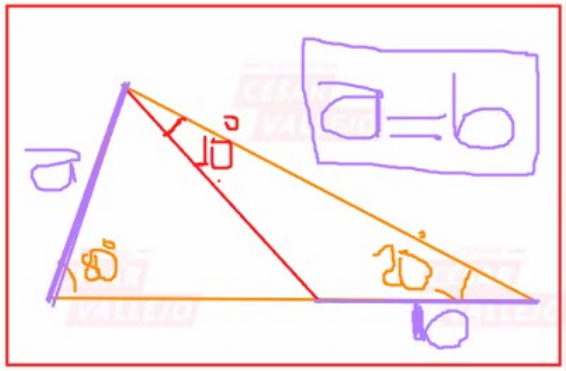
Problema 3

En el gráfico, $AD=BC$. Calcule x.

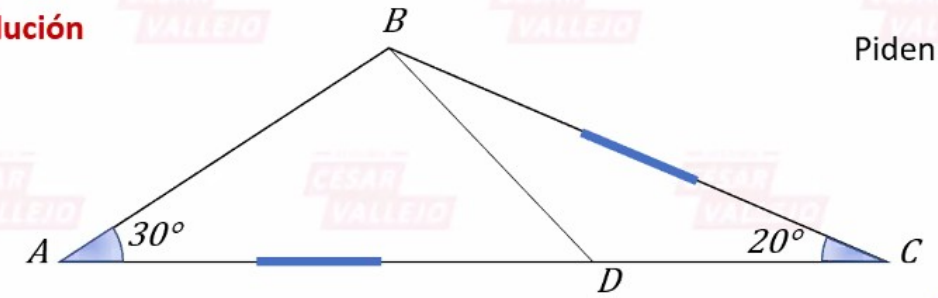
A) 60°
B) 50°
C) 70°
D) 40°
E) 30°





Resolución

Piden:



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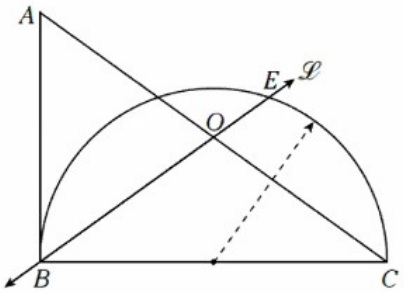
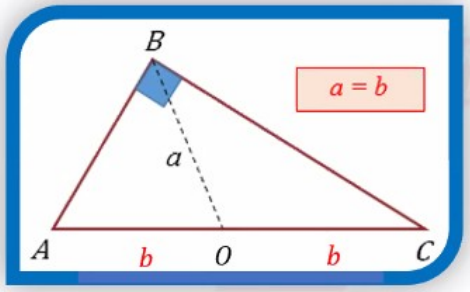
Problema 4

En el gráfico $\overleftrightarrow{\mathcal{L}}$ es la recta de Euler correspondiente al triángulo ABC. Si $BO=3(OE)=3$, calcule BC.



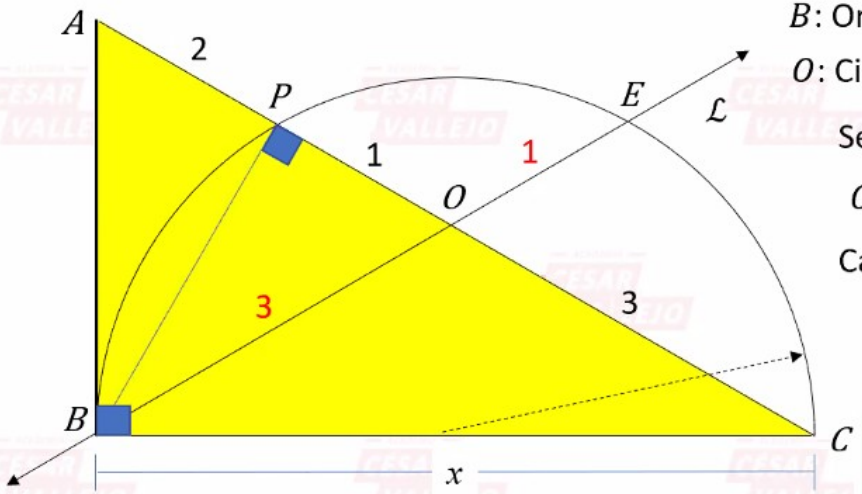
A) $2\sqrt{6}$ B) $\sqrt{6}$ C) $3\sqrt{2}$
D) 5 E) $2\sqrt{3}$

Si O es circuncentro:



Resolución

Piden: $BC = x$



Como $\overleftrightarrow{\mathcal{L}}$ es la recta de Euler, entonces:

B: Ortocentro del triángulo ABC

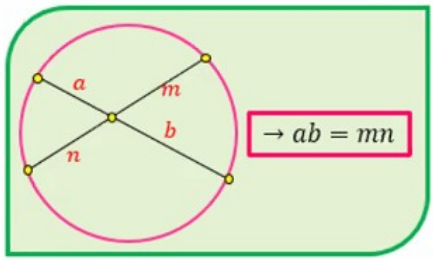
O: Circuncentro del triángulo ABC

Se sigue que:

$$OC = \boldsymbol{OA} = BO = 3$$

Calculemos PO:

TEOREMA DE LAS CUERDAS



Por el teorema de cuerdas: $(OB)(OE) = (OC)(PO)$

➡ $OP = 1$, y $AP = 2$

Trazamos $\overline{BP}$

$\measuredangle ABC$: $BC^2 = (CA)(CP)$

$x^2 = (6)(4)$

Finalmente: $x = 2\sqrt{6}$

CLAVE: **A**

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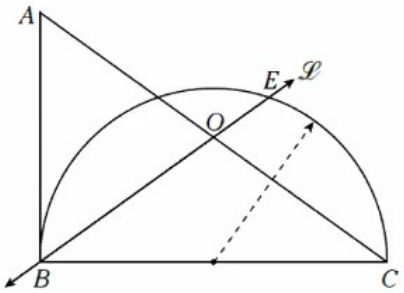
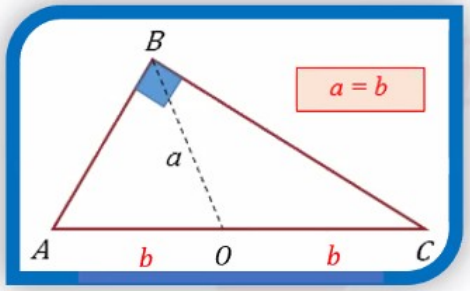
Problema 4

En el gráfico $\overleftrightarrow{\mathcal{L}}$ es la recta de Euler correspondiente al triángulo ABC. Si $BO=3(OE)=3$, calcule BC.



A) $2\sqrt{6}$ B) $\sqrt{6}$ C) $3\sqrt{2}$
D) 5 E) $2\sqrt{3}$

Si O es circuncentro:



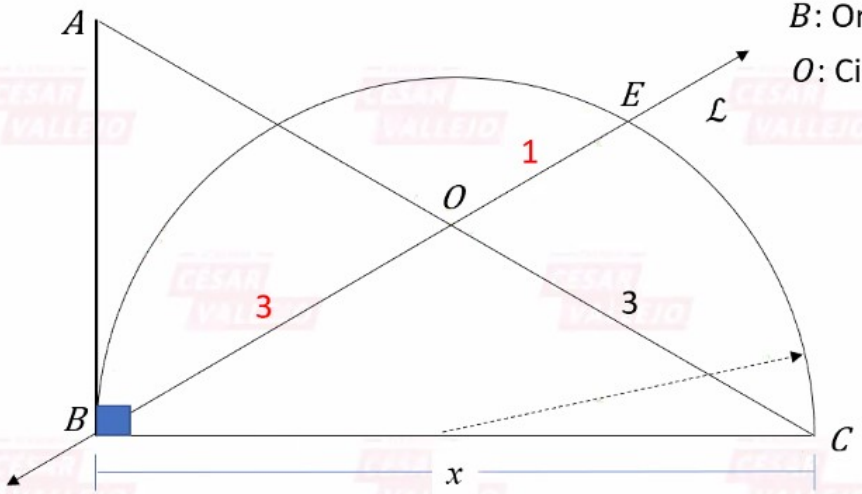
Resolución

Piden: $BC = x$

Como $\overleftrightarrow{\mathcal{L}}$ es la recta de Euler, entonces:

B: Ortocentro del triángulo ABC

O: Circuncentro del triángulo ABC



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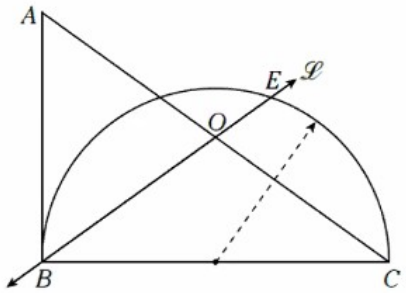
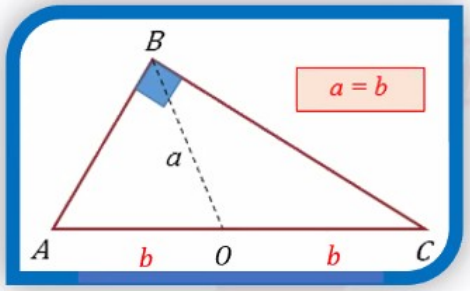
Problema 4

En el gráfico $\overleftrightarrow{\mathcal{L}}$ es la recta de Euler correspondiente al triángulo ABC. Si $BO=3(OE)=3$, calcule BC.



A) $2\sqrt{6}$ B) $\sqrt{6}$ C) $3\sqrt{2}$
D) 5 E) $2\sqrt{3}$

Si O es circuncentro:



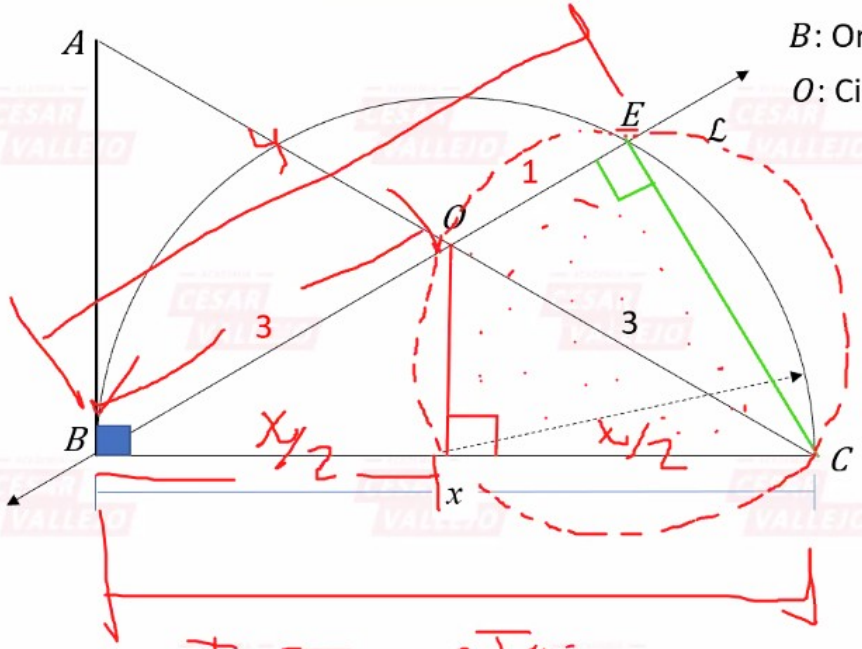
Resolución

Piden: $BC = x$

Como $\overleftrightarrow{\mathcal{L}}$ es la recta de Euler, entonces:

B: Ortocentro del triángulo ABC

O: Circuncentro del triángulo ABC



T. SECUNTO1

$x \cdot \frac{x}{2} = 4 \times 3$

$x^2 = 24$

$\therefore x = 2\sqrt{6}$

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Problema 6

Según el gráfico, $ABCD$ es un cuadrado, en el lado $\overline{BC}$ se ha trazado un semicírculo. Si $BM=a$ y $MN=b$, calcule NC.

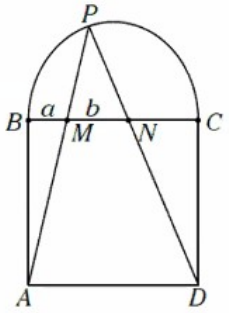
A) $\frac{b}{a}$

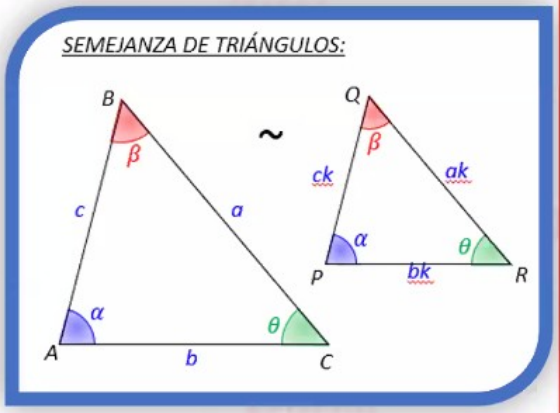
B) $\frac{b^2}{a}$

C) a

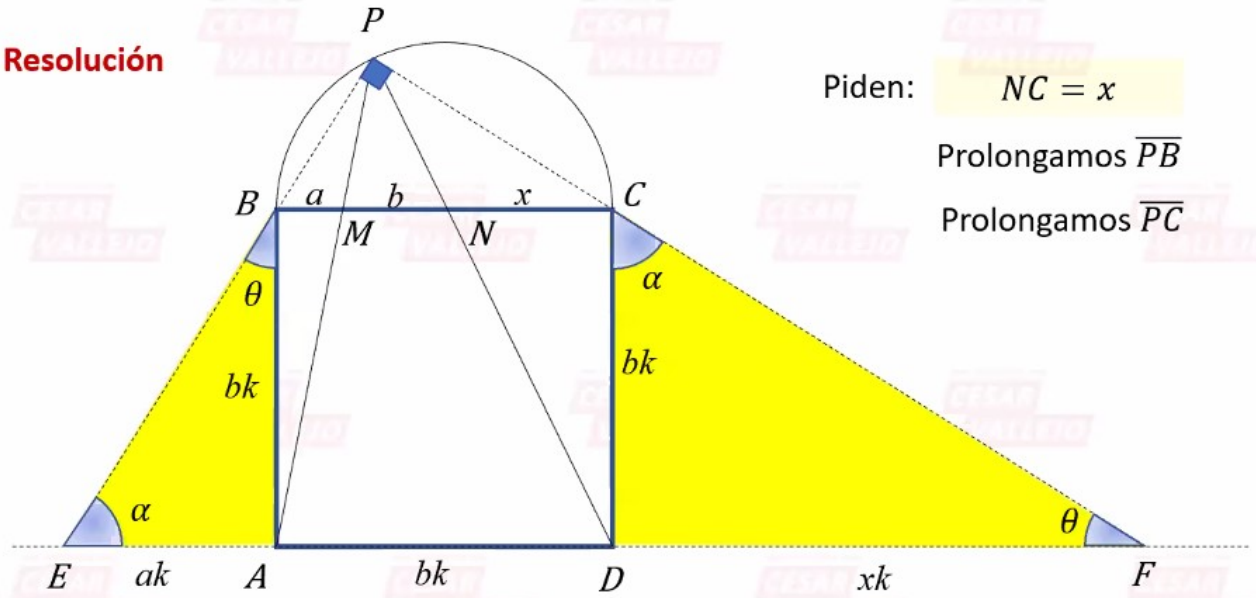
D) b

E) $\frac{a+b}{2}$





Resolución



Piden: $NC = x$

Prolongamos $\overline{PB}$

Prolongamos $\overline{PC}$

Construimos el triángulo rectángulo EPF

Ya que $\overline{BC} \parallel \overline{EF}$, tendremos la semejanza: $\Delta BPC \sim \Delta EPF$

Usando elementos homólogos:

$$\frac{EA}{a} = \frac{AD}{b} = \frac{DF}{x} \quad \begin{cases} EA = ak \\ AD = bk \\ DF = xk \end{cases}$$

Completamos medidas angulares $\Delta EAB \sim \Delta CDF$

$$\frac{ak}{bk} = \frac{bk}{xk} \Rightarrow x = \frac{b^2}{a}$$

CLAVE: **B**

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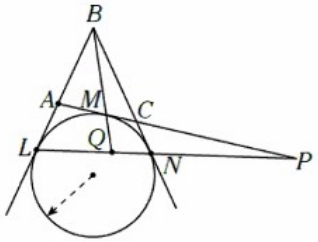
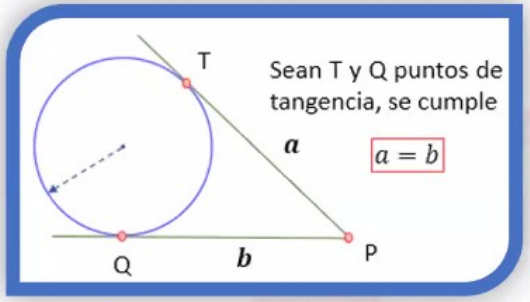
Problema 7

Si L, M y N son puntos de tangencia, además, $AM=a$ y $MC=b$. Calcule CP.

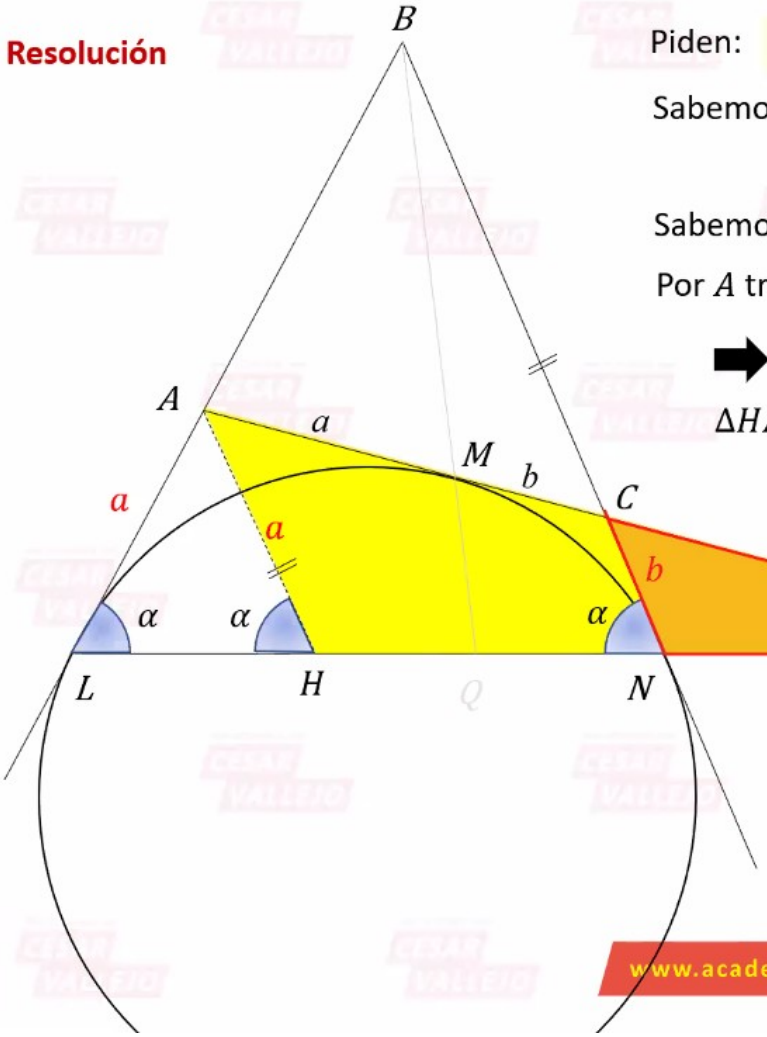


A) $a+b$ B) $\sqrt{ab}$ C) $\left(\frac{a+b}{2}\right)b$

D) $\frac{(a+b)b}{(a-b)}$ E) $\frac{(a+b)a}{(a-b)}$



Resolución



Piden: $CP = x$

Sabemos que $AL = AM = a$

$CN = CM = b$

Sabemos que: $m\measuredangle BLN = m\measuredangle BNL = \alpha$

Por A trazamos $\overline{AH}$ la paralela a $\overline{CN}$

$\Rightarrow m\measuredangle AHL = \alpha$

$\Delta HAP \sim \Delta NCP$

$$\frac{x}{x+a+b} = \frac{b}{a}$$

Finalmente:

$$x = \frac{(b+a)b}{a-b}$$

CLAVE: **D**

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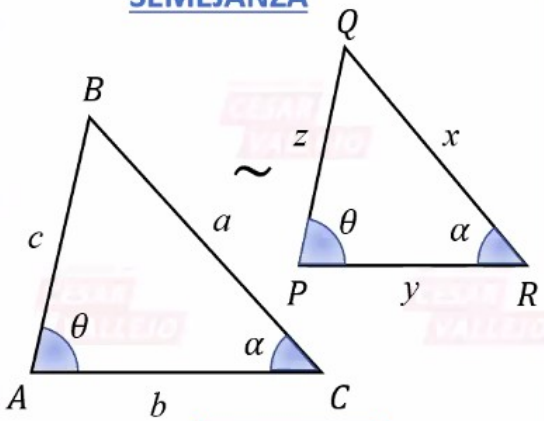
Problema 8

En una circunferencia se inscribe el triángulo equilátero ABC, la cuerda $\overline{AE}$ interseca al lado BC en el punto F. Si $BF=4$ y $AD=12$, calcule la longitud del lado del triángulo ABC si $\overrightarrow{BE} \cap \overrightarrow{AC} = \{D\}$.

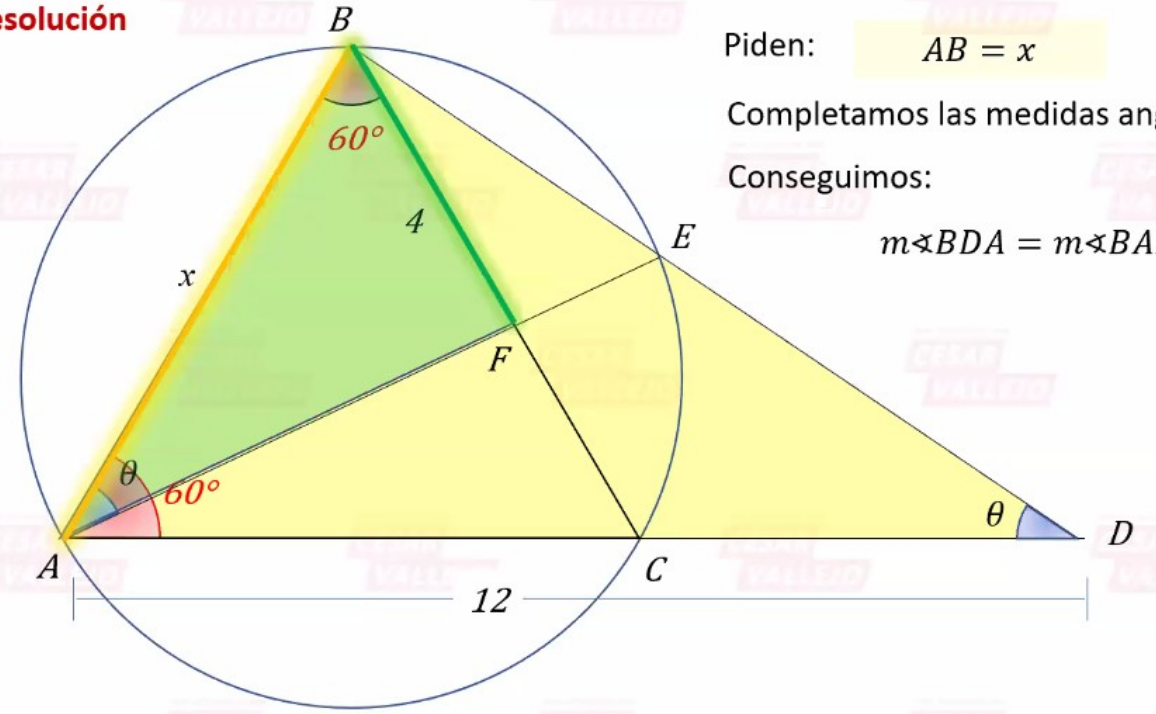
A) $4\sqrt{2}$ B) $5\sqrt{2}$ C) $4\sqrt{3}$
D) 8 E) $9\sqrt{5}$

SEMEJANZA



$$\frac{a}{x}=\frac{b}{y}=\frac{c}{z}$$

Resolución



Piden: $AB = x$

Completamos las medidas angulares

Conseguimos:

$$m\sphericalangle BDA = m\sphericalangle BAE = \theta$$

$\Delta ABF \sim \Delta DAB$ $\quad \frac{12}{x}=\frac{x}{4}$

Finalmente: $x = 4\sqrt{3}$

CLAVE: **C**

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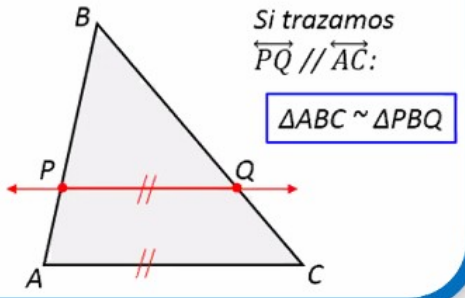
Problema 10

Dos circunferencias congruentes y tangentes exteriores cuyos radios miden 2, están inscritos en un triángulo ABC de modo que ambas circunferencias son tangentes al lado $\overline{AC}$, una de ellas es tangente al lado $\overline{AB}$ y la otra a $\overline{BC}$, si $AC=16$. Halle la longitud del inradio del triángulo ABC.

A) $\frac{3}{4}$ B) $\frac{5}{4}$ C) $\frac{8}{7}$

D) $\frac{8}{3}$ E) $\frac{6}{5}$

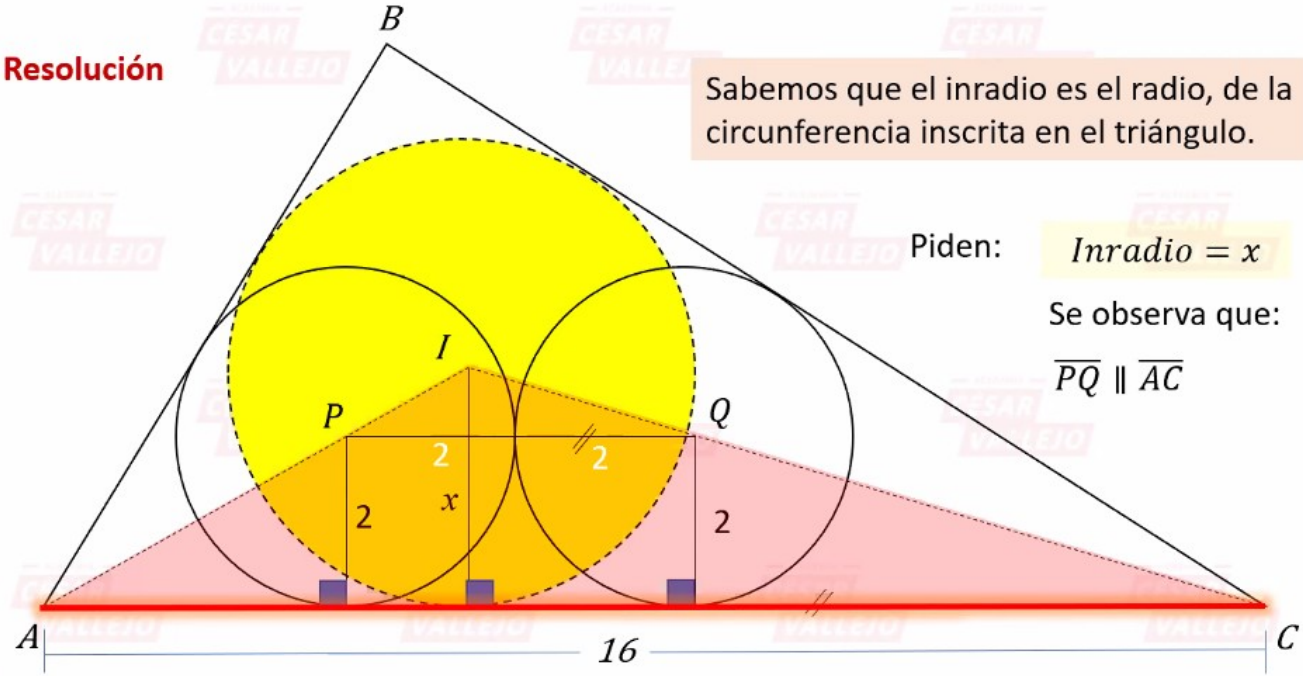
SEMEJANZA:



Si trazamos $\overleftrightarrow{PQ} // \overleftrightarrow{AC}$:

$\Delta ABC \sim \Delta PBQ$

Resolución



Sabemos que el inradio es el radio, de la circunferencia inscrita en el triángulo.

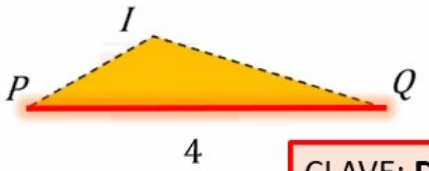
Piden: $Inradio = x$

Se observa que:

$\overline{PQ} \parallel \overline{AC}$

$\Delta AIC \sim \Delta PIQ$

$$\frac{x}{x-2} = \frac{16}{4}$$



Operando:

$$x = \frac{8}{3}$$

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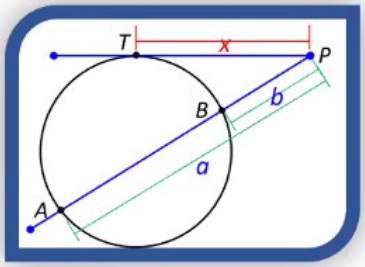
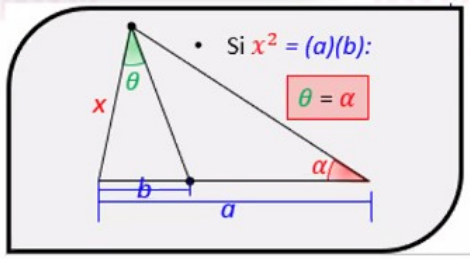
Problema 11

Desde un punto P exterior a una circunferencia, se trazan $\overline{PA}$ y $\overline{PB}$ tangentes a la circunferencia (A y B son puntos de tangencia), en el triángulo ABP se traza la mediana $\overline{BM}$ que interseca a la circunferencia en el punto N. Si $MN=2$, calcule NP.

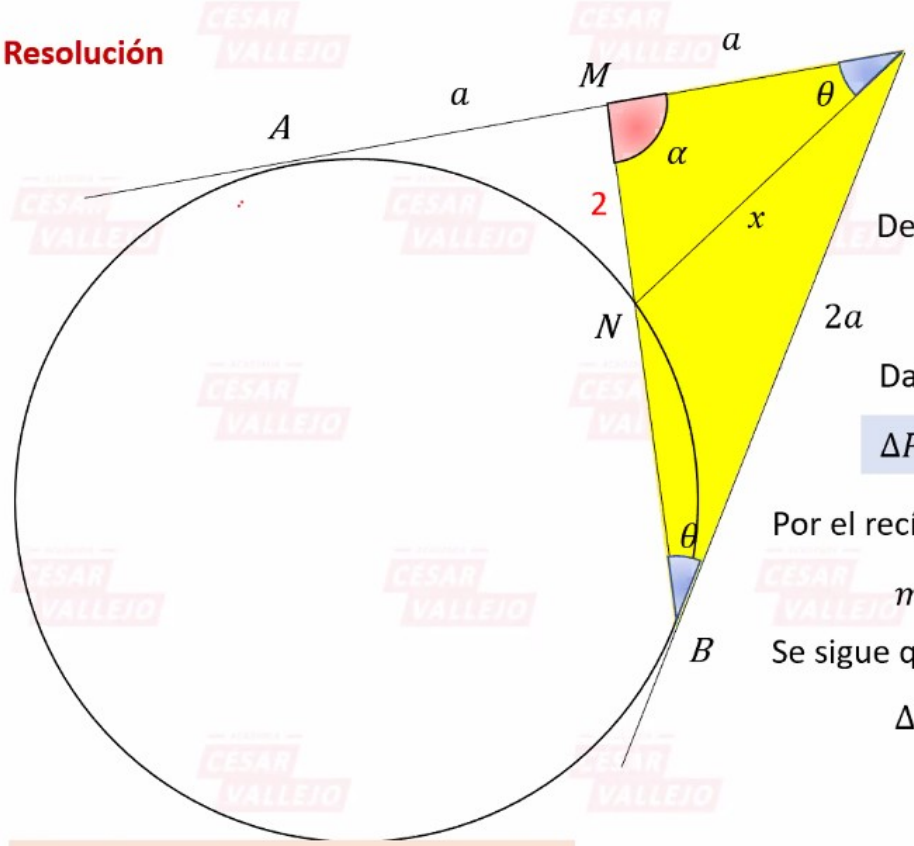
A) 5 B) 1 C) 3
D) 2 E) 4

Teorema de la tangente





Resolución



Piden:

$NP = x$

Del teorema de la tangente:

$a^2 = 2(MB)$

Dato: $MP = MA = a$

ΔPMB: $a^2 = 2(MB)$

Por el recíproco de las anti paralelas

$m\measuredangle MPN = m\measuredangle MBP$

Se sigue que:

$\Delta MPN \sim \Delta MBP$

Recordemos que: $PA = PB = 2a$

$$\frac{x}{2a} = \frac{2}{a} \Rightarrow x = 4$$

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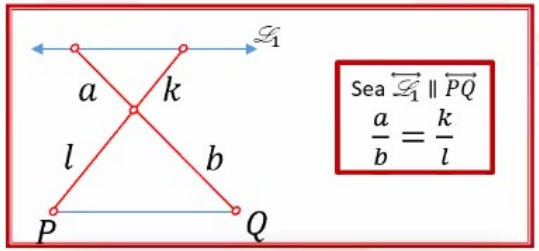
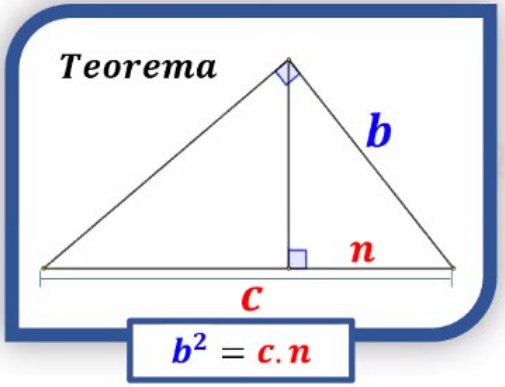
Problema 12

En un paralelogramo $ABCD$, se ubica M en $\overline{BC}$ y Q en $\overline{CD}$, $(\overline{AM} \cap \overline{BD} = \{L\})$, $m\sphericalangle AMQ = m\sphericalangle LQD = 90°$, si $AL = 6$. Calcule LQ.

A) $2\sqrt{6}$ B) $4\sqrt{2}$ C) 4
D) $3\sqrt{2}$ E) 6

Teorema:





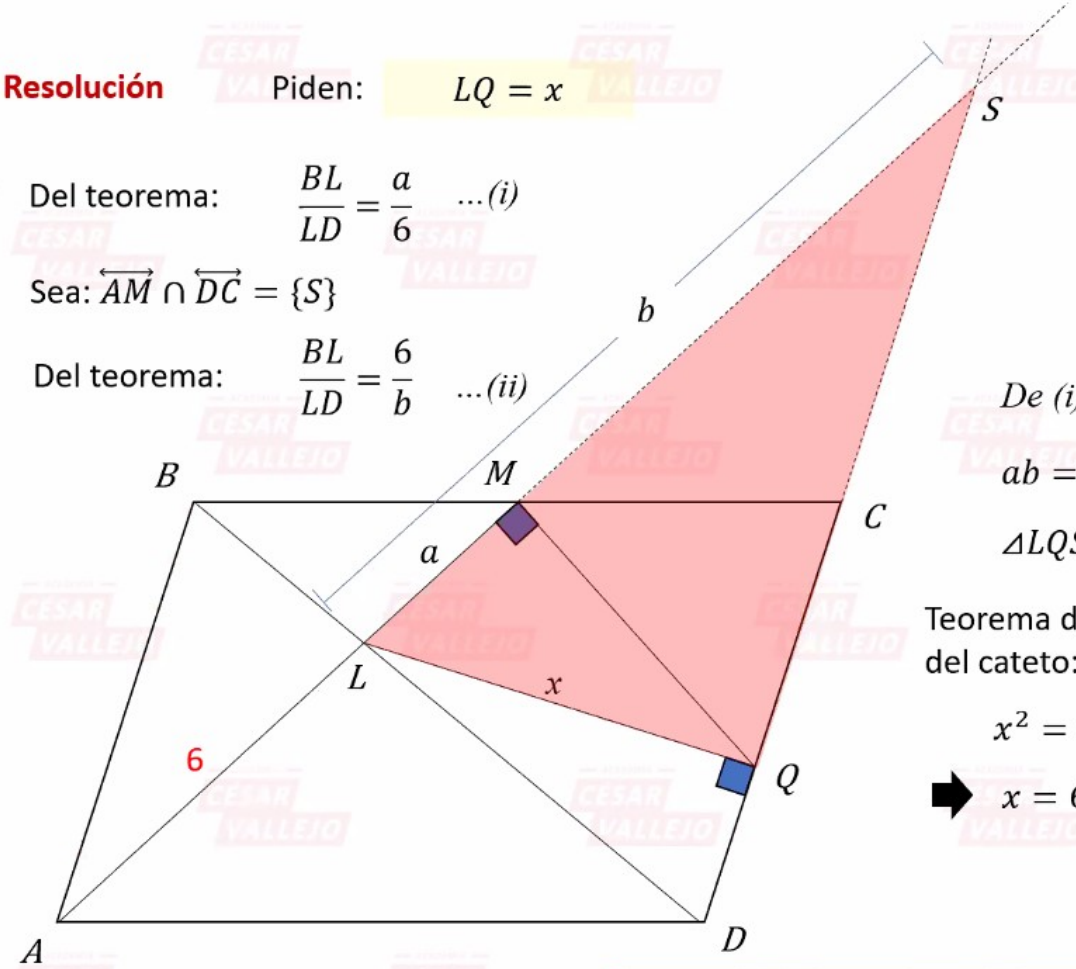
Resolución

Piden: $LQ = x$

Del teorema: $\frac{BL}{LD} = \frac{a}{6}$ ...(i)

Sea: $\overleftrightarrow{AM} \cap \overleftrightarrow{DC} = \{S\}$

Del teorema: $\frac{BL}{LD} = \frac{6}{b}$ ...(ii)



De (i) y (ii):

$ab = 36$

$\triangle LQS$:

Teorema del cuadrado del cateto:

$x^2 = ab$

$\Rightarrow x = 6$

CLAVE: **E**

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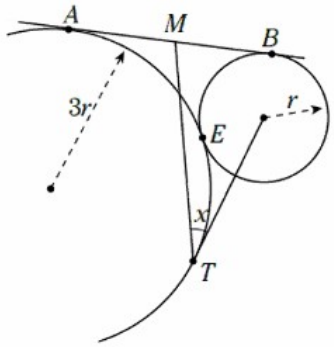
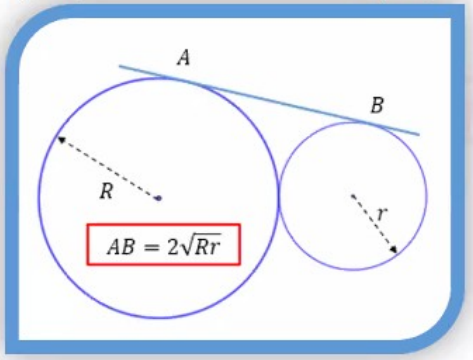
Problema 14

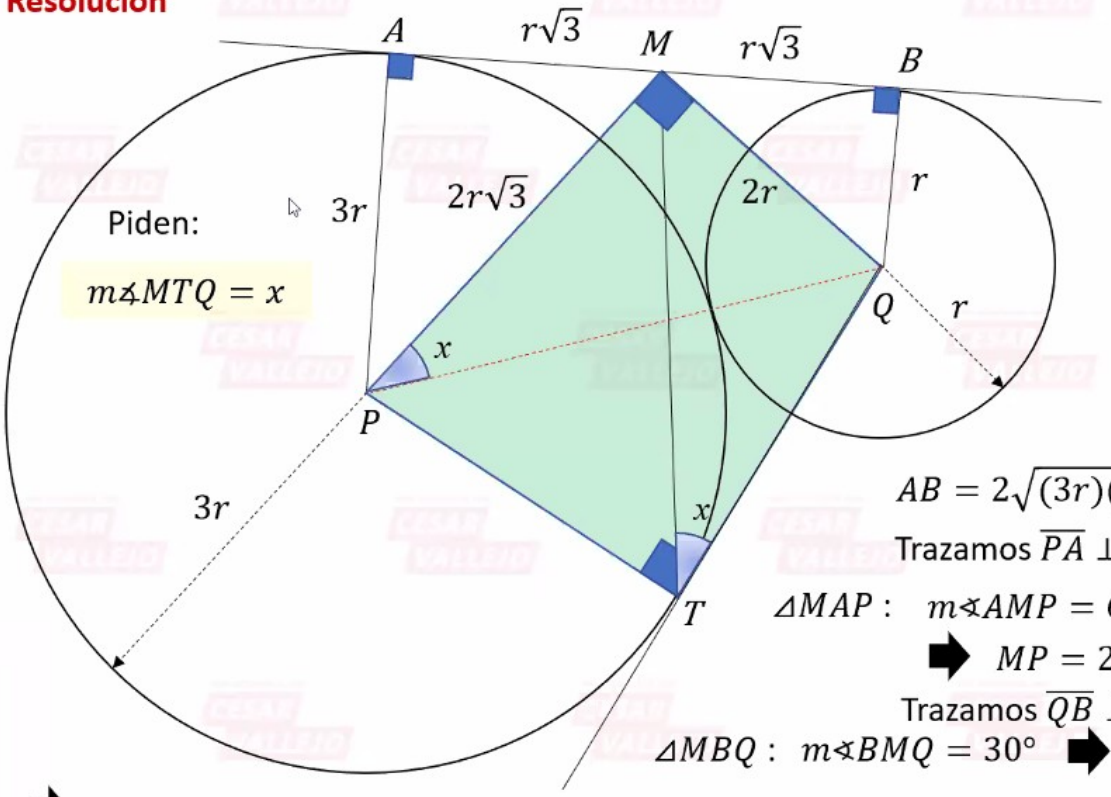
Según el gráfico A, B; E y T son puntos de tangencia. $AM=MB$. Calcule x.



A) 30° B) 45° C) 16°
D) 37°/2 E) 53°/2



Resolución



Piden: $m\measuredangle MTQ = x$

Entonces:

$AB = 2\sqrt{(3r)(r)} = 2r\sqrt{3}$

Trazamos $\overline{PA} \perp \overline{AB}$

$\triangle MAP$: $m\measuredangle AMP = 60°$

➡ $MP = 2r\sqrt{3}$

Trazamos $\overline{QB} \perp \overline{AB}$

$\triangle MBQ$: $m\measuredangle BMQ = 30°$ ➡ $MQ = 2r$

➡ $m\measuredangle PMQ = 90°$

Trazamos $\overline{PT}$: $MQTP$: Inscriptible ➡ $m\measuredangle MPQ = m\measuredangle MTQ = x$

$\triangle PMQ$: $x = 30°$

CLAVE: **A**

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